Technical Notes

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Vortex Theory for Hovering Rotors

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CLASSICAL vortex theories as originally developed for propellers assumed wake-induced velocities small compared with the forward velocity, an assumption that permitted such geometric simplification as modeling the spiral wake by a semi-infinite vortex cylinder of constant strength and diameter whose properties are well known. A similar model applied to the case of a hovering helicopter gives results in agreement with momentum theory. Yet for this case the assumptions used are not consistent with the properties of the model, nor are the spanwise airload distributions in agreement with test data. The purpose of this Note is to review these inconsistencies and to show that fairly simple vortex models that do include wake contraction give results in reasonable agreement both with momentum theory and with test data.

Gray[1] first demonstrated experimentally that the actual wake geometry of a hovering rotor differs appreciably from the assumption of classical vortex theory. Further experimental investigations[2,3] of rotor wake geometry led to development of a reliable prescribed wake model for computing blade airloads. Theoretical techniques were also developed for computing wake geometries given a prescribed blade airload distribution.[4,5] The availability of more capable computers permitted the development of unconstrained free-wake analytical techniques in which neither the airloads nor the wake geometry is prescribed a priori and their interdependence is permitted.[6-10] In these models the wake is generally represented by vortex filaments or sheets, the induced velocities are computed everywhere in the wake, and the blade airloads are related to the resulting wake geometry. Such solutions agree reasonably well with the observed results, but they require extensive computer analyses and are difficult to use for heuristic or formal optimization. There is also strong evidence that relatively complex real-fluids effects are of importance for the close blade vortex interactions typical of rotor blade aerodynamics.[7,14] For these reasons a simpler model, but one that retains the basic essentials of the free-wake analytical techniques, would be of help in understanding the complex physics of the problem and could serve as a guide to more elaborate aerodynamic modeling.

The nature of the problem will be illustrated by a very brief review of classical rotor vortex theory. If, for simplicity, it is assumed that a rotor blade has a twist distribution such that the bound circulation Γ is a constant (ideal twist), then the wake will consist primarily of a vortex spiral generated by the bound vortex leaving the blade at the tip. This spiral, when approximated by a semi-infinite vortex cylinder, has a strength along the zR axis given by

$$\frac{d\Gamma}{dz} = \frac{\Gamma}{2\pi}\frac{Q}{w/\Omega R}$$

where R is the rotor radius, w the axial velocity in the wake, ΩR the rotor tip speed, and Q the number of blades. For a hovering rotor, $w \ll \Omega R$.

The induced velocity at the blade will first be determined by neglecting wake contraction and assuming w is independent of z. Applying the Biot-Savart law, the vertical component of induced velocity at any radial station ηR along the blade is

$$w_I = \frac{Q}{8\pi^2 w/\Omega R}\frac{\Gamma}{R}\int_0^\infty\int_0^{2\pi}\frac{1-\eta\cos\phi}{(1+\eta^2+z^2-2\eta\cos\phi)^{3/2}}\,d\phi\,dz \tag{1}$$

Integration first with respect to z results in

$$w_I = \frac{Q}{8\pi^2 w/\Omega R}\frac{\Gamma}{R}\int_0^{2\pi}\frac{1-\eta\cos\phi}{1+\eta^2-2\eta\cos\phi}\,d\phi \tag{2}$$

When $\eta=1$, that is, at the vortex cylinder, the integral has the value π. When $\eta\neq 1$, the integration may be performed by a change of variable $e^{i\phi}=x$ and application of the residue theorem. When $\eta<1$, the integral has the value 2π; when $\eta>1$, it is 0.

The vertical component of induced velocity over the rotor disk is therefore uniform and twice the velocity at the cylinder. The induced velocity at a point in the fully developed wake, $z\to\infty$, is obtained by integration from $z=\infty$ to $-\infty$, giving the result that the induced velocity at infinity is twice that at the rotor disk.

The total rotor thrust is

$$T = QR^2\Gamma\int_0^1 \rho\Omega\eta\,d\eta \tag{3}$$

In classical propeller vortex theory, when the induced velocity is a perturbation on the total axial velocity through the rotor disk due to forward flight, a reasonable assumption, consistent with the neglect of wake contraction, would be $w=w_I$. Defining $\lambda=w/\Omega R$ and $C_T=T/\rho\pi R^2\Omega^2R^2$, it follows from Eqs. (2) and (3) that $C_T=2\lambda^2$, which is the same result as that obtained from momentum theory for the hovering rotor.

It has been established previously, however, that the induced velocity at the vortex cylinder, $\eta=1$, is $w_I/2$. In hovering flight, where the only axial velocity is the induced velocity, this must be the velocity that determines at least the initial strength of the vortex cylinder, $d\Gamma/dz$. Evidently, if $w=w_I/2$, the value of C_T computed from vortex theory would not agree with the well-established results from momentum theory. It is therefore necessary to review the assumptions made in developing the preceding solution:

1a) Wake contraction may be neglected.

2a) The wake spacing is determined by the velocity over the rotor disk rather than the velocity at the edge of the disk.

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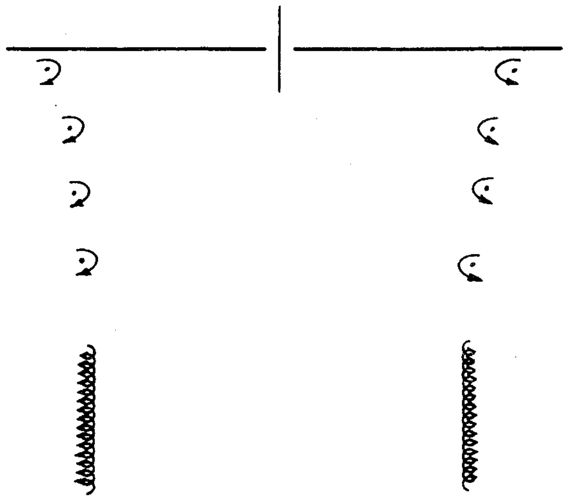

Fig. 1 Side view of simplified wake model.

3a) The strength $d\Gamma/dz$ of the vortex cylinder is independent of z.

Although these assumptions may be reasonable in the case of a propeller in forward flight, they are clearly inadmissible for the hovering rotor, since the well-known theoretical results summarized previously indicate that

1b) The velocity in the fully developed wake is twice that at the rotor disk, and hence the vortex cylinder must contract.

2b) The velocity at the vortex cylinder is half the velocity over the rotor disk.

3b) It follows from 1b that $d\Gamma/dz$ must vary with z.

The properties 1b-3b are confirmed by experimental data, as originally demonstrated in Ref. 1. One is therefore faced with a paradox in that the assumptions necessary to develop a vortex theory for the hovering rotor which agrees with momentum theory are not consistent with the properties of the vortex model used (semi-infinite vortex cylinder) or with the test data.

It is for this reason that the free-wake analytical techniques previously discussed, free of the constraints of classical propeller theory, are required. It will be shown that very simple models, for example, a quasi-two-dimensional one in which velocities are computed only at points in the wake below the blade, are adequate for this purpose. The rotor wake is replaced by: 1) a near wake consisting of semi-infinite vortex filaments attached to the blade in the plane of the rotor; 2) pairs of discrete infinite line vortices below the rotor blade, forming an intermediate wake; and 3) a far wake consisting of a pair of vortex sheets. This geometry is possible because of the symmetry of the rotor in hovering flight. Viewed from the side, this wake model appears as a two-dimensional model (Fig. 1).

The velocity induced by the vortex sheets is

$$w_l = \frac{l}{2\pi R}\frac{d\Gamma}{dz}\int_z^\infty \left[\frac{l-\eta}{(l-\eta)^2+z^2} + \frac{l+\eta}{(l+\eta)^2+z^2}\right]dz$$

and the integral has the values

$$\pi - \tan^{-l}\frac{z}{l-\eta} - \tan^{-l}\frac{z}{l+\eta} \qquad \eta < r$$

$$\pi/2 - \tan^{-l}\frac{z}{l+\eta} \qquad \eta = r$$

$$\tan^{-l}\frac{z}{l-\eta} - \tan^{-l}\frac{z}{l+\eta} \qquad \eta > r$$

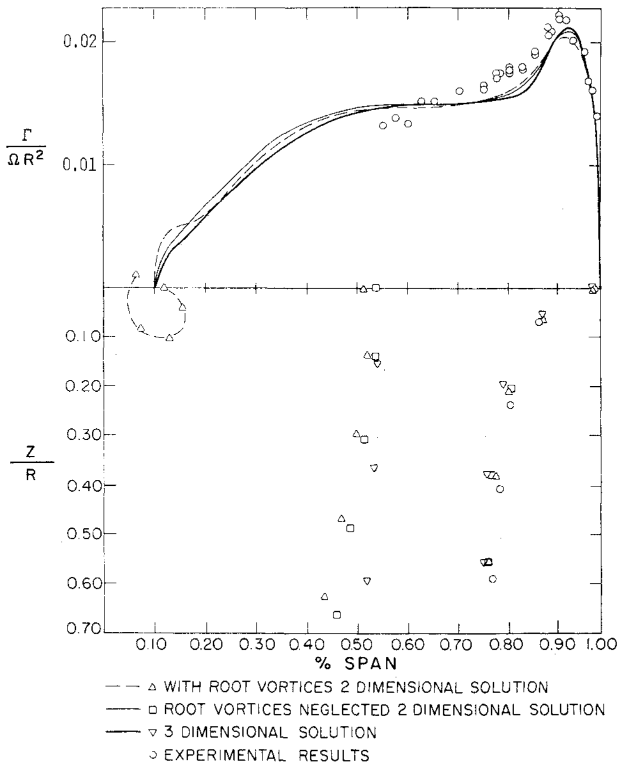

Fig. 2 Blade bound circulation distribution and location of wake vortices.

It may be verified readily by setting $z=0$ that this model gives the same results as the vortex cylinder model of Eq. (1) for assumptions 1a-3a listed previously.

In Ref. 11 free-wake solutions based on this two-dimensional (2-D) model and a three-dimensional (3-D) model using vortex rings and cylinders were used to compute the wake geometry and bound circulation distribution for comparison with the experimental results of Ref. 12 and the estimated-wake positions of Ref. 13. Solutions were obtained for a four-bladed rotor[14] and compared with the experimental results of Ref. 15.

In all these solutions it was assumed that the near wake rolled up into vortex filaments, conserving linear momentum in the process, according to various schedules determined by the bound circulation distribution. The comparisons for the two-bladed rotor are shown in Fig. 2, taken from Ref. 16, for the case of two and three vortex filaments and for the 2-D and the 3-D solutions. Evidently the bound circulation on the blade is heavily influenced by wake contraction. Neither the inflow nor the circulation distributions are close to the almost uniform values predicted by classical vortex theories. The 2-D and 3-D models gave essentially similar results despite the simplification involved in arriving at the 2-D model. The measured bound circulation gave a C_T of 0.00459. The computed C_T for the 2-D case was 0.00456. Without the root vortex C_T was 0.00460, and C_T for the corresponding 3-D case was 0.00452. Momentum theory gave a C_T of 0.00442. Reference 16 contains a more detailed discussion of the theoretical development as well as the program description and listing for both models.

Conclusion

The apparent paradox resulting from application of the classical propeller vortex theory to a hovering rotor results from the neglect of wake contraction. When wake contraction is included, simple models are capable of predicting blade load distribution and wake geometry to a reasonable degree of accuracy.

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Oblique Impingement of a Round Jet on a Plane Surface

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Introduction

THE oblique impingement of a round jet on a plane surface establishes a three-dimensional flowfield with properties of relevance to the analysis of VTOL aircraft in-ground effect. The azimuthal distribution of momentum flux resulting from such an impact cannot be found solely on the basis of global momentum theorems (e.g., Taylor[1]). Truncated Fourier series models,[2] as well as detailed numerical analyses,[3] have been used to represent this distribution for VTOL applications. The former technique provides useful estimates, while the latter technique, though correct in principle, suffers from computational resolution problems and the inherent inability to provide a simple, easily used formula.

Direct momentum efflux measurements of a three-dimensional submerged, jet impingement flowfield are made difficult to achieve by the effects of entrainment.[4] Taylor[5] used colliding water jets to measure directly the efflux distribution for jet inclinations of $\theta=\pi/6$, $\pi/4$, and $\pi/3$ and uncovered several global features of the flow. He noted that, within the thin collision sheet, the flow was very nearly radial from some point within the impact region, except perhaps at the most shallow of inclinations (i.e., $\theta=\pi/6$). Many years earlier, Schach[6] measured the upstream and downstream efflux resulting from round water jets impinging upon a plane wall. Over the entire range of incidence angle (viz., $\theta=\pi/6$, $\pi/4$, $\pi/3$, $5\pi/12$, and $\pi/2$), he found that his data was in excellent agreement with a rather simple model that he had devised.

The purpose of this Note is to present a simple expression for the azimuthal redistribution of impingement momentum flux. This expression in deduced from the Schach model and accurately predicts the observations made by Taylor.

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Schach's Model

Schach considered a uniform, inviscid and incompressible jet of unit radius impinging with angle θ upon a plane surface (Fig. 1). The stagnation streamline resides in the symmetry plane and, away from the stagnation point, it is parallel to the jet axis separated by the distance b. The intersection of the stagnation line with a plane normal to the jet axis (viz., the jet plane) forms the origin for angle β, where β is measured from the symmetry plane. Flow through the region $-\pi/2<\beta<\pi/2$ is deflected downstream and flow through the region $\pi/2<\beta<3\pi/2$ is deflected upstream. If the distance from the origin to the jet edge is denoted by $r(\beta,b)$, then

$$r(\beta,b) = b\cos\beta + \sqrt{1-b^2\sin^2\beta} \tag{1}$$

and the momentum flux through the elemental area included by $d\beta$ is

$$dJ = \tfrac{1}{2}\rho u^2 r^2(\beta,b)\,d\beta$$

where ρ and u are the jet density and velocity, respectively, and the jet momentum flux J is

$$J = \rho u^2 \int_0^\pi r^2(\beta,b)\,d\beta = \pi\rho u^2$$

The efflux far from the stagnation point recovers to the jet velocity and is assumed to spread along surface rays defined by the azimuthal angle ϕ. Here, ϕ is measured from the symmetry plane with origin at the stagnation point (Fig. 1). The ray $\phi=\pm\pi/2$ divides the efflux into its upstream ($x<0$) and downstream components. Schach reasoned that, since the jet plane forms an angle $(\pi/2-\theta)$ with the surface plane (Fig. 2), the angle β undergoes a change to angle $\phi(\beta)$ given by

$$\tan\phi = \sin\theta\tan\beta \tag{2}$$

Conservation of x momentum demands that

$$J\cos\theta = 2\int_0^\pi \cos\phi\,dJ = \frac{J}{\pi}\int_0^\pi r^2(\beta,b)\cos\{\phi(\beta)\}\,d\beta \tag{3}$$